Kingdom of Saudi Arabia
Ministry of Education
College of Computer
Department of Computer Engineering

المملكة العربية السعودية
وزارة التعليم
كلية الحاسب
قسم هندسة الحاسب

Course Code and Name: MATH218 & Differential Equations
MID Exam
461 - Date: 29/10/2024 Duration: 1 hour

| Total Marks |
| --- |
| 30 |

Student Name: Student ID: Group: S.N.:

Answer All Questions

أسلوب الحل: أحل كل واحد

1. Find the differential equation $xy'-2y=1+y'$. (2 marks)

**Solution:**

The D.E $xy'-2y=1+y'$ is a clairsaut form

So, the solution is $xc-2y=1+c$

$$\boxed{xc-2y=1+c}$$

Clairaut's eq

2. Solve the differential equation $\frac{dy}{dx}=e^{3x-5y}$ separation of variables. (5 Marks)

سؤال أساسي لازم يجي

**Solution:**

$$\frac{dy}{dx}=e^{3x}e^{-5y}$$

$$e^{5y}dy=e^{3x}dx$$ , Separable Variable

Integrate id

$$\int e^{5y}dy=\int e^{3x}dx$$

$$\frac{e^{5y}}{5}=\frac{e^{3x}}{3}+C$$

$$\frac{dy}{dx}=e^{3x}e^{-5y}$$

$$e^{5y}dy=e^{3x}dx$$

$$\int e^{5y}dy=\int e^{3x}dx$$

$$\frac{e^{5y}}{5}=\frac{e^{3x}}{3}+C$$

3. Determine whether the function $f(x) = \sqrt{x^2 + y^3}$ is homogeneous. If so, state the degree of homogeneity. (2 marks)

**Solution:**

أكيد بجيب

The Power of each term are different

So, $f(x) = \sqrt{x^2+y^3}$ is not H.G nogenous.

$f(x) = \sqrt{t^2x^2 + t^3y^3} \neq t^n f(x,y)$

∴ it's not H.G

سحش يم

4. Solve the second order differential equation $y'' + 2y' + 2y = 0$. (5 Marks)

**Solution:**

Sec 4.3

$y'' + 2y' + 2y = 0$

$m^2 + 2m + 2 = 0$

$a=1, b=2, c=2$

$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

A.E. is $m^2 + 2m + 2 = 0$

$m = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2}$

$m = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$

$m = \frac{-2 \pm \sqrt{-4}}{2}$ → $\sqrt{4}\,i$

$m = \frac{-2 \pm 2i}{2}$

$m = -1 \pm i$

The solution is

$y = e^{\alpha x}[C_1 \cos\beta x + C_2 \sin\beta x]$

$y = e^{-x}[C_1 \cos x + C_2 \sin x]$

$m = -1 \pm i$

case 3: m1 and m2 are complex numbers

Then $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$

case 3 ↓ فيه 3 حالات

مع التكرار بعيد سهل الحل

Sol:

$y = e^{\alpha x}[C_1 \cos\beta x + C_2 \sin\beta x]$

$y = e^{-x}[C_1 \cos x + C_2 \sin x]$

كان جزا تحديد الميد ج

5. Find the differential operator that annihilates $x^2e^{-3x} + e^{2x}\sin 3x$. **(5 Marks)**

**Solution:**

$x^2e^{-3x}$

$(D-\alpha)^n x^{n-1} e^{\alpha x} = 0$

$\alpha = -3, \quad n = 3$

$(D+3)^3 x^2 e^{-3x} = 0$

$e^{2x}\sin 3x$

$[D^2 - 2\alpha D + (\alpha^2+\beta^2)]^n e^{\alpha x}\sin\beta x = 0$

$\alpha = 2, \quad \beta = 3, \quad n = 1$

$(D^2 - 4D + 13)^1 e^{2x}\sin 3x = 0$

Then, $(D+3)^3(D^2-4D+13)$ is Differential operator that annihilates $x^2e^{-3x} + e^{2x}\sin 3x$

6. Determine whether the given equation is exact. If exact, Solve $(y^2x + 3x^2)dx + (x^2y + 2\cos y)dy = 0$. **(5 Marks)**

سؤال سهل ويتكرر دايم

**Solution:**

$M = y^2x + 3x^2, \quad N = x^2y + 2\cos y$

$\frac{\partial M}{\partial y} = 2xy, \quad \frac{\partial N}{\partial x} = 2xy.$

So, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

The solution is $\int_{y=\text{const}} M\,dx + \int N\,dy = C$

Term free from $x$

$\int (y^2x + 3x^2)\,dx + \int 2\cos y\,dy = C.$

$\frac{y^2x^2}{2} + x^3 + 2\sin y = C$

$$M = y^2x + 3x^2 \quad , \quad N = x^2y + 2\cos y$$

$$\frac{\partial M}{\partial y} = \frac{y^2x}{\partial y} + \frac{3x^2}{\partial y}$$

$$\frac{\partial M}{\partial y} = 2yx + 6$$

$$\frac{\partial N}{\partial x} = \frac{x^2y}{\partial x} + \frac{2\cos y}{\partial x}$$

$$\frac{\partial N}{\partial x} = 2yx + 0$$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ∴ it's exact

$$\boxed{\int M\,dx + \int N\,dy = C} \rightarrow \int y^2x + 3x^2\,dx + 2\int \cos y\,dy$$

y const — Term free from x

$$\frac{y^2x^2}{2} + x^3 + 2\sin y = C$$

...battery is connected to a series circuit in which he inductance is 1/4 henry and the ...resistance is 10 ohms. Determine the current i if the initial current is zero. (6 Marks)

**Solution:** R L

The diff. eq$^{n}$. of the circuit is

$$L\frac{di}{dt} + Ri = E(t)$$

i is current

given $L = \frac{1}{4}$ ← Inductance, $R = 10$ ← Resistance, $E = 5$ ← volt

$$\frac{1}{4}\frac{di}{dt} + 10i = 5$$

$$\frac{di}{dt} + 40i = 20 \quad \text{(1)}$$

$P(t) = 40$

I.f. $= e^{\int P(t)dt} = e^{40t}$ ← نضربه بكل حد

from (1)

$$e^{40t}\frac{di}{dt} + 40ie^{40t} = 20e^{40t}$$

$$\frac{d}{dt}(ie^{40t}) = 20e^{40t}$$

now Integrate: $ie^{40t} = 20\int e^{40t}dt = \frac{20}{40}e^{40t}$

$$i(t) = \frac{1}{2} + ce^{-40t}$$

given $t = 0$, $i = 0$

$$0 = \frac{1}{2} + c \Rightarrow c = -\frac{1}{2}$$

So, $\boxed{i(t) = \frac{1}{2} - \frac{1}{2}e^{-40t}}$

E volt

L Inductance

R Resistance

C → Capacitance

هذا السكشن هو بداية الأصعبة بين وبينك
ما أظن جابه أبد لكن احفظ القانون احتياطي

12 volt series circuit Inductance $\frac{1}{2}$

R is 10 current, is zero

$$L\frac{di}{dt}+Ri=E(t) \quad (1)$$

$$\frac{1}{2}\frac{di}{dt}+10i=12(t) \rightarrow \frac{di}{dt}+\overset{20}{2\cdot 10}=\overset{24}{2\cdot 12}$$

(2) $P(t)=20$, $If=e^{\int P(t)dt}=e^{20t}$

$$e^{20x}\frac{di}{dt}+e^{20x}20=e^{20x}24$$

Integrate:

$$\frac{d}{dt}(ie^{20x})=24e^{20x} \rightarrow ie^{20x}=24\int e^{20t}dt$$

$$i e^{20t} = 24 \frac{e^{20t}}{20} + C \rightarrow i = \frac{6}{5} \frac{e^{20t}}{e^{20t}} + \frac{C}{e^{20t}}$$

$$i(0) = \frac{6}{5} + C e^{-20t}$$

(2)

given initial value is zero

$i(0) = 0 \rightarrow t = 0 , i = 0$ $\quad e^0 = 1$

From (2) $i(0) = \frac{6}{5} + C \; e^0$

$\Rightarrow 0 = \frac{6}{5} + C$

$\Rightarrow C = -\frac{6}{5} \rightarrow$ From (2) $i(t) = \frac{6}{5} - \frac{6}{5} e^{-20t}$

**Answer the following questions.**

1. Evaluate $ydx + e^{3x}dy = 0$ by separation of variables. **(6 Marks)**

2. Solve the linear differential equation $\frac{dy}{dx} + y = e^{3x}$ **(6 Marks)**

3. Determine whether the given equation is exact. If exact, **(6 Marks)**
   Solve $(sinx + y^2)dx + (2yx + cosy)dy = 0$.

4. Evaluate $y'' + 2y' + 2y = 0$. **(6 Marks)**

5. Find the second solution of the differential equation $y'' - 4y' + 4y = 0$; $y_1 = e^{2x}$. Assume an appropriate interval of validity. **(6 Marks)**

1

$ydx + e^{3x}dy = 0$

Solu

$ydx = -e^{3x}dy \Rightarrow -\frac{dx}{e^{3x}} = \frac{dy}{y} \Rightarrow -e^{-3x} = \ln y \Rightarrow \frac{e^{-3x}}{3} = \ln y + C$

$\frac{dy}{dx} + y = e^{3x}$

Solu

$P(x) = 1 \quad \mu = e^{\int 1dx} = e^x$

$e^x\frac{dy}{dx} + ye^x = e^{4x}$

$\frac{d}{dx}(ye^x) = e^{4x}$

$\frac{dy}{dx}[e^x y] = e^{4x}$

$\int \frac{d}{dx}(ye^x)dx = \int e^{4x}dx$

$\int \frac{dy}{dx}[e^x y]dx = \int e^{4x}dx$

$ye^x =$

3

$(\sin x + y^2)dx + (2yx + \cos y)dy = 0$

Solu

$M = \sin x + y^2$ & $N = 2yx + \cos y$

$\frac{\partial M}{\partial y} = 2y$ | $\frac{\partial N}{\partial x} = 2y$

It is Exact

$$\int M\,dx + \int N\,dy = C$$

y is constant

$$\int \sin x + y^2\,dx + \int \cos y\,dy \quad \text{(free of x)}$$

$$\boxed{-\cos x + xy^2 + \sin y = C}$$

4

$y'' + 2y' + 2y = 0$

Solu

$A.E = m^2 + 2m + 2$

$m = -1 \pm i$

$\alpha \pm \beta i$

$y = e^{\alpha x}[C_1 \cos \beta x + C_2 \sin \beta x]$

$$\boxed{y = e^{-1}[C_1 \cos x + C_2 \sin x]}$$

(5) $y'' - 4y' + 4y = 0$ , $y_1 = e^{2x}$

Solu

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2} dx$$

$$P(x) = -4$$

$$e^{-\int P(x)dx} = e^{+\int 4 dx} = e^{4x}$$

$$y_2 = e^{2x} \int \frac{e^{4x}}{(e^{2x})^2} dx$$

$$y_2 = e^{2x} \int \frac{e^{4x}}{e^{4x}} dx$$

$$y_2 = e^{2x} \int dx$$

$$y_2 = e^{2x} \cdot x + c$$

$$\boxed{y_2 = x e^{2x} + c}$$