بسم الله الرحمن الرحيم

Kingdom of Saudi Arabia
Ministry of Education
College of Computer
Computer Science Department

المملكة العربية السعودية
وزارة التعليم
كلية الحاسب
قسم علوم الحاسب

MATH212: Discrete Mathematics **Midterm Exam. 13/10/2024**

Time: 1 hour **25 Marks**

Name: **ID:**

Question1: ***Answer with TRUE or FALSE, justify your answers:*** ***(5 M)***

a) The propositional function $\neg(\forall x\, P(x))$ is equivalent to $\neg\, \exists x$ such that $P(x)$)

False, justification: $\neg(\forall x\, P(x)) \equiv \exists x \neg (P(x))$

b) If the propositional function P(x) is given by $(x^2 = 4) \rightarrow (x \leq 1)$. Then P (2) is true.

False, justification: T → F

c) If the domain consists of all real numbers, then $\forall x(x^2 \neq x)$

False, Justification: $x = 0$ and $x = 1$ satisfy $x^2 = x$, contradicting the statement.

d) $\emptyset \subseteq \{x\}$

True, justification: the empty set is a subset of every set.

e) $\left|P\left(\{\emptyset, a, \{a\}, \{\{a\}\}\}\right)\right| = 8$

False, Justification: the cardinality of the sets $2^4 = 16$.

Question 2: ***Given the following compound propositions:*** ***(5+1 M)***

$a)\ (p \rightarrow q) \wedge r$, b) $p \rightarrow (\neg r \vee q)$

1. Construct a truth table for the compound propositions a and b.
2. Are the compound propositions a and b logically equivalent?

p	q	r	p → q	(p → q) ∧ r	¬ r	¬ r ∨ q	p → (¬ r ∨ q)
T	T	T	T	T	F	T	T
T	T	F	T	F	T	T	T
T	F	T	F	F	F	F	F
T	F	F	F	F	T	T	T
F	T	T	T	T	F	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	F	F	T
F	F	F	T	F	T	T	T

a and b are not logically equivalent.

Question 3: *(3 M)*

Let P(x) be the statement "x can speak English" and let Q(x) be the statement "x knows the computer language Python" Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

1. There is a student at your school who can speak English but who doesn't know Python.
   Solution: $\exists x\,(P(x) \wedge \neg Q(x))$
2. Every student at your school either can speak English or knows Python.
   Solution: $\forall x\,(P(x) \vee Q(x))$
3. No student at your school can speak English or knows Python.
   Solution: $\neg \exists x\,(P(x) \vee Q(x))$ or $\forall x \neg (P(x) \vee Q(x))$

Question 4: ***Give a contrapositive proof of the following:*** *(4 M)*

If $x^2 + 3$ is even, then x is odd, for $x \in \mathbb{Z}$

We assume that p(x) : $x^2 + 3$ is even , q(x) : x is odd

Proof. (Contrapositive):

Suppose x is not odd. Thus x is even, and $x = 2k$ for some integer k.

Hence, $x^2 + 3 = (2k)^2 + 3 = 4k^2 + 3 = 4k^2 + 2 + 1 = 2(2k^2 + 1) + 1$

Therefore $x^2 + 3 = 2r + 1$, where r is the integer equal to $2k^2 - 6k + 2$.

Consequently $x^2 - 6x + 5$ is odd.

That is, if $\neg q \rightarrow \neg p$ true, then $p \rightarrow q$ true, which proof that if $x^2 - 6x + 5$ is even, then x is odd.

Question 5: ***Let A = {a, b, c, 2, 3}, B = {c, d, 3}, and C = {1, 2}, with the universal set U={1, 2, 3, a, b, c, d}. Find:*** *(7 M)*

a) $B \times C$
b) C^2
c) $(B \cap C) \cup A$
d) $\overline{A \cup B} \cap C$
e) C ∩ (B − C)
f) $P(C)$
g) $|A \cup B|$

a) $B \times C = \{(c,1),(c,2),(d,1),(d,2),(3,1),(3,2)\}$
b) $C^2 = C \times C = \{(1,1),(1,2),(2,1),(2,2)\}$
c) $(B \cap C) \cup A = \phi \cup \{a,b,c,2,3\} = \{a,b,c,2,3\}$
d) $\overline{A \cup B} \cap C = \{1\} \cap \{1,2\} = \{1\}$
e) $C \cap (B - C) = \{1,2\} \cap \{c,d,3\} = \phi$
f) $P(C) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$
g) $|A \cup B| = |A| + |B| - |A \cap B| = 5 + 3 - 2 = 6$

تمنياتنا لكم بالتوفيق والسداد