Kingdom of Saudi Arabia
Ministry of Education
College of Computer
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المملكة العربية السعودية
وزارة التعليم
كلية الحاسب
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Course Code and Name: MATH218 and Differential Equations
Mid-Term Exam
462 - Date: 4/03/2025 Duration: 1 Hour

Total Marks
30

Student Name:	Student ID:

Answer all questions:

1. Evaluate $e^y x\cos x dx + y dy = 0$ by separation of variables. **(5 Marks)**

Solution:

$$e^y x\cos x\,dx = -y\,dy$$

$$x\cos x\,dx = -\frac{y}{e^y}\,dy$$

$$x\cos x\,dx = -y e^{-y}\,dy \quad \text{(Separable Variable}$$

Integrate it

$$\int x\cos x\,dx = -\int y e^{-y}\,dy$$

$$x\sin x + \cos x = -\left[y\frac{e^{-y}}{-1} - 1\cdot e^{-y}\right] = y e^{-y} + e^{-y} + C.$$

2. Solve the linear differential equation $\frac{dy}{dx} + y = e^{2x}$. —— (1) **(5 Marks)**

Solution: Compare with $\frac{dy}{dx} + P(x)y = f(x)$.

$$P = 1, \quad I.f. = e^{\int P(x)dx} = e^{\int dx} = e^x$$

Multiply (1) by $I.f. = e^x$

$$e^x\frac{dy}{dx} + e^x y = e^{3x}$$

$$\frac{d}{dx}(ye^x) = e^{3x}$$

$$y e^x = \int e^{3x}dx$$

$$y e^x = \frac{e^{3x}}{3} + C$$

$$y = \frac{e^{2x}}{3} + Ce^{-x}$$

3. Solve homogeneous differential equation $(x+2y)dx+xdy=0$. **(5 Marks)**

Solution: Put $y=ux \Rightarrow dy = u\,dx + x\,du$

$(x+2ux)dx + x(u\,dx + x\,du) = 0$

$x\,dx + 2ux\,dx + xu\,dx + x^2\,du = 0$

$(1+3u)x\,dx + x^2\,du = 0$

$$\frac{x\,dx}{x^2} + \frac{du}{1+3u} = 0$$

Integrate, $$\frac{dx}{x} = -\frac{du}{1+3u}$$

$$\ln x = -\frac{1}{3}\ln(1+3u) = -\frac{1}{3}\ln(1+3y/x) + C.$$

4. Determine whether the given equation is exact. If exact, Solve $(y^2+\cos x)dx+(2xy+\sin x)dy=0$. **(5 Marks)**

Solution: Compare with $M\,dx + N\,dy = 0$

$M = y^2 + \cos x$, $N = 2xy + \sin x$

$$\frac{\partial M}{\partial y} = 2y, \qquad \frac{\partial N}{\partial x} = 2y + \cos x$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So, given differential equation is not exact.

5. Solve $y''+3y'+3y=0$. **(5 Marks)**

Solution: Auxiliary Equation is

$$m^2+3m+3=0$$

$$m=\frac{-3\pm\sqrt{9-4\times3}}{2}=\frac{-3\pm\sqrt{-3}}{2}=\frac{-3\pm\sqrt{3}i}{2}$$

$$=-\frac{3}{2}\pm\frac{\sqrt{3}}{2}i$$

The Solution is

$$y=e^{-\frac{3}{2}x}\left[C_1\cos\frac{\sqrt{3}}{2}x+C_2\sin\frac{\sqrt{3}}{2}x\right].$$

6. Show by computing the Wronskian that the function e^{-3x}, e^{2x} is linearly independent on the interval $(-\infty,\infty)$. **(5 Marks)**

Solution:

Let $y_1=e^{-3x}$, $y_2=e^{2x}$

$$W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}=\begin{vmatrix} e^{-3x} & e^{2x} \\ -3e^{-3x} & 2e^{2x} \end{vmatrix}$$

$$=2e^{-x}+3e^{-x}$$

$$=+5e^{-x}\neq0$$

so, given functions are linearly Independent.