المملكة العربية السعودية
وزارة التعليم العالي
كلية الحاسب
قسم علوم الحاسب
الرقم الجامعي
اسم الطالب:

Question1: ***Answer with TRUE or FALSE.***

a) If 25 is divisible by 5 then 40 is divisible by 7. (F)

b) If the domain consists of all integers, then $\forall n\,(n^2 > n)$ (F)

c) If a set B contains 4 elements, then the number elements of $P(B)$ is 8. (F)

d) $S = \{x \mid x$ is a positive integer less than $100\}$ is a set builder Notation of representation of sets. (T)

e) Let A and B finite sets and $f: A \to B$ a function then $|f(A)| \leq |A|$. (T)

Question 2 : ***Given the following compound propositions:*** **(6 M)**

(a): $(p \wedge q) \to r$, (b): $(p \to r) \vee (q \to r)$

(i) Construct a truth table for the compound propositions (a) and (b).

(ii) Are the compound propositions (a) and (b) logically equivalent?

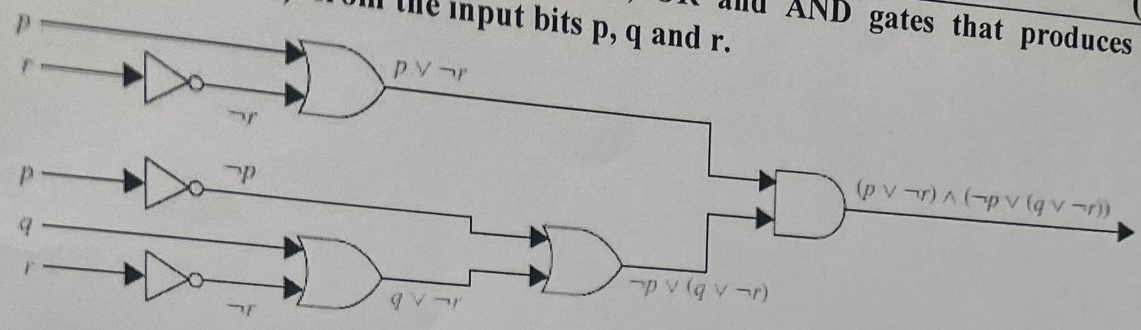
(i):

P	q	r	$p \wedge q$	$(p \wedge q) \to r$	$p \to r$	$q \to r$	$(p \to r) \vee (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	T
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

(ii): Hence, from the truth table (a) and b are logically equivalent.

Question 3:

Construct a combinatorial circuit using inverters, OR and AND gates that produces the output $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$ from the input bits p, q and r. **(4 M)**



Question 4:

(3 M)

Let P(x) be the statement "x can speak Russian" and let Q(x) be the statement "x knows the computer language C++." Express the sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian but who doesn't know C++.

$$\exists x \left(P(x) \wedge \neg Q(x)\right)$$

b) Every student at your school either can speak Russian or knows C++.

$$\forall x \left(P(x) \vee Q(x)\right)$$

d) No student at your school can speak Russian or knows C++.

$$\neg \exists x \left(P(x) \vee Q(x)\right) \equiv \forall x \neg \left(P(x) \vee Q(x)\right)$$

Question 5:

(5 M)

Prove that if n is an integer and $7n + 4$ is even, then n is even using a proof by contraposition.

Let p(n): $7n + 4$ is even number and q(n): n is even.

An implication $p \rightarrow q$ *is equivalent to its contrapositive* $\neg q \rightarrow \neg p$,

Suppose n is odd, and $n = 2k + 1$ for some integer k. Hence,

$$7n + 4 = 7(2k + 1) + 4 = 14k + 7 + 4 = 14k + 11 = 14k + 10 + 1 = 2(7k + 5) + 1 = 2r + 1$$

where $r = 7k + 5$ is an integer. Consequently, $7n + 4$ is odd.

That is, if $\neg q \rightarrow \neg p$ true, then $p \rightarrow q$ true, which proof that if $7n + 4$ is even number, then n is even number.

tion 6: (7 M)

$= \{2, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{2, 4\}$, with the universal set $\{1, 2, 3, \dots, 10\}$. Find:

) $A \times C =$

) $C^2 =$

) $(B \cap C) \cup A =$

d) $\overline{A \cup B} \cap C =$

e) $C \cap (B - C) =$

f) $P(C) =$

g) $|A \cup B| =$

a) $A \times C = \{(2,2), (2,4), (4,2), (4,4), (7,2), (7,4), (10,2), (10,4)\}$
b) $C^2 = C \times C = \{(2,2), (2,4), (4,2), (4,4)\}$
c) $(B \cap C) \cup A = \{2,4\} \cup \{2,4,7,10\} = \{2,4,7,10\} = A$
d) $\overline{A \cup B} \cap C = \{6,8,9\} \cap \{2,4\} = \{\}$
e) $C \cup (B - C) = \{2,4\} \cup \{1,3,5\} = \{1,2,3,4,5\} = B$
f) $P(C) = \{\emptyset, \{2\}, \{4\}, \{2,4\}\}$
g) $|A \cup B| = |A| + |B| - |A \cap B| = 4 + 5 - 2 = 7$