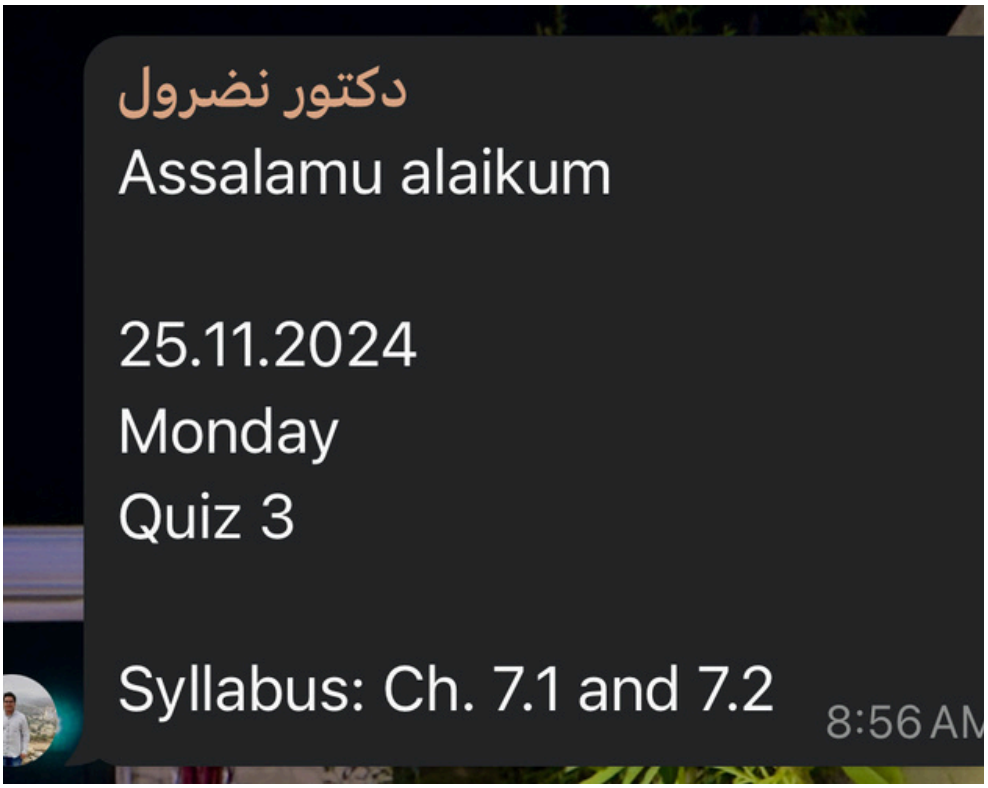
MATH218

Quiz 3

462

التحديد :



Kingdom of Saudi Arabia
Ministry of Education
College of Computer
Department of Computer Engineering

Qassim University — College of Computer

المملكة العربية السعودية
وزارة التعليم
كلية الحاسب
قسم هندسة الحاسب

Course Code and Name: MATH218 & Differential Equations
Quiz 3
461 - Date: 27/11/2024 Duration: 10 Min

Total Marks
5

Student Name: Student ID: S.N.:

Answer all questions:

1. Evaluate $L[3 + t^2]$ (1 Mark)

Solution: $F(t) = 3 + t^2$

$L[F(t)] = L(3) + L(t^2)$ using $L(1) = \frac{1}{s}$, $L(t^n) = \frac{n!}{s^{n+1}}$

$= \frac{3}{s} + \frac{2!}{s^3}$ note: $2! = 2$

Aseen_XX3

2. Find $L[f(t)]$ if $f(t) = \sin 3t + e^{-3t}$ (2 Marks)

Solution:

$L[F(t)] = L(\sin 3t) + L(e^{-3t})$ using $L(\sin Kt) = \frac{K}{s^2+K^2}$, $L(e^{at}) = \frac{1}{s-a}$, $L(e^{-at}) = \frac{1}{s+a}$

$= \frac{3}{s^2+3^2} + \frac{1}{s+3}$

3. Evaluate $L^{-1}\left[\frac{1}{s^2+2} + \frac{5}{s}\right]$ (2 Mark)

$L[F(t)] = F(s)$
$L^{-1}[F(s)] = F(t)$

Solution:

$F(t) = L^{-1}\left(\frac{1}{s^2+2}\right) + L^{-1}\left(\frac{5}{s}\right)$ using $1 = L^{-1}\left(\frac{1}{s}\right)$, $\sin Kt = L^{-1}\left[\frac{K}{s^2+K^2}\right]$

$= L^{-1}\left(\frac{1}{s^2+(\sqrt{2})^2}\right) + 5L^{-1}\left(\frac{1}{s}\right)$

$= \frac{1}{\sqrt{2}} L^{-1}\left(\frac{\sqrt{2}}{s^2+(\sqrt{2})^2}\right) + 5(1)$

let $2 = (\sqrt{2})^2 \rightarrow \frac{\sqrt{2}}{s^2+(\sqrt{2})^2} = \left[\frac{K}{s^2+K^2}\right]$
and add $\sqrt{2}$ to 1 → divide by $\frac{1}{\sqrt{2}}$ to balance it

$= \frac{1}{\sqrt{2}} \sin\sqrt{2}t + 5$

7.2 Inverse Laplace Transform

Laplace Transform	Inverse Laplace Transform
(a) $\mathcal{L}(1) = \frac{1}{s}$	(a) $1 = \mathcal{L}^{-1}\left(\frac{1}{s}\right)$
(b) $\mathcal{L}(t^n) = \frac{n!}{s^{n+1}}$, $n = 1, 2, \cdots$	(b) $t^n = \mathcal{L}^{-1}\left[\frac{n!}{s^{n+1}}\right]$
(c) $\mathcal{L}(e^{at}) = \frac{1}{s-a}$	(c) $e^{at} = \mathcal{L}^{-1}\left(\frac{1}{s-a}\right)$
(d) $\mathcal{L}(\sin kt) = \frac{k}{s^2+k^2}$	(d) $\sin kt = \mathcal{L}^{-1}\left[\frac{k}{s^2+k^2}\right]$
(e) $\mathcal{L}(\cos kt) = \frac{s}{s^2+k^2}$	(e) $\cos kt = \mathcal{L}^{-1}\left[\frac{s}{s^2+k^2}\right]$
(f) $\mathcal{L}(\sinh kt) = \frac{k}{s^2-k^2}$	(f) $\sinh kt = \mathcal{L}^{-1}\left[\frac{k}{s^2-k^2}\right]$
(g) $\mathcal{L}(\cosh kt) = \frac{s}{s^2-k^2}$	(g) $\cosh kt = \mathcal{L}^{-1}\left[\frac{s}{s^2-k^2}\right]$

$$\mathcal{L}[f(t)] = F(s)$$

$$\Rightarrow f(t) = \mathcal{L}^{-1}[F(s)]$$