Kingdom of Saudi Arabia
Ministry of Education
College of Computer
Department of Computer Engineering

Qassim University
College of Computer | كلية الحاسب
جامعة القصيم

Course Code and Name: MATH218 & Differential Equations
MIDTERM Exam (B)
472 - Date: 12/04/2026 Duration: 1 hour

Total Marks
30

Student Name: **Student ID:** **Group:** **S.N.:**

Answer all questions: 6X5 Marks

1. Solve $\frac{dx}{dy} = \frac{1+2y^2}{xysinx}$ by separation of variables.

2. Find the differential operator that annihilates $x^2e^{2x} + e^{-x}cos2x$.

3. Sove $y'' + y' + 2y = 0$.

4. Solve the Bernoulli equation $x\frac{dy}{dx} + y = \frac{1}{y^2}$.

5. Show by computing the Wronskian that the functions $f_1 = x^3$ and $f_2 = 1 - x$ are linearly independent on the $(-\infty, \infty)$.

6. Determine whether the given equation is exact. If exact,
Solve $(ycosx + 2x)dx + (2y + sinx)dy = 0$.

Solution

1. $\frac{dx}{dy} = \frac{1+2y^2}{xy\sin x}$

$x\sin x\,dx = \frac{1+2y^2}{y}\,dy$

Integrate $\int x\sin x\,dx = \int \frac{1}{y}dy + 2\int y\,dy$

$x(-\cos x) - 1\cdot(-\sin x) = \ln y + y^2 + c$

$-x\cos x + \sin x = \ln y + y^2 + c$

2. $x^2e^{2x} + e^{-x}\cos 2x$

$(D-\alpha)^n x^{n-1}e^{\alpha x} = 0$ | $[D^2 - 2\alpha D + \alpha^2 + \beta^2]^n x^{n-1}e^{\alpha x}\cos\beta x = 0$

$x^2e^{2x} = 0$ | $\alpha = -1,\ n = 1,\ \beta = 2$

$n = 3,\ \alpha = 2$ | $(D^2 + 2D + 5)^1 e^{-x}\cos 2x = 0$

$(D-2)^3 x^2 e^{2x} = 0$

So, $(D-2)^3 (D^2 + 2D + 5)(x^2e^{2x} + e^{-x}\cos 2x) = 0$

Thus, $(D-2)^3(D^2+2D+5)$ annihilates $(x^2e^{2x} + e^{-x}\cos 2x)$.

3. $y'' + y' + 2y = 0$

A.E. is $m^2 + m + 2 = 0$

$$m = \frac{-1 \pm \sqrt{1-8}}{2}$$

$$m = -\frac{1}{2} \pm \frac{\sqrt{7}}{2}i$$

$$y = e^{-\frac{1}{2}x}\left[c_1 \cos\frac{\sqrt{7}}{2}x + c_2 \sin\frac{\sqrt{7}}{2}x\right].$$

5. $f_1 = x^3,\ f_2 = 1-x$

$$W = \begin{vmatrix} x^3 & 1-x \\ 3x^2 & -1 \end{vmatrix} = -x^3 - 3x^2 + 3x^3 = 2x^3 - 3x^2 \neq 0$$

So, f_1 and f_2 are L.I. in $(-\infty, \infty)$

4. $x\frac{dy}{dx} + y = \frac{1}{y^2}$

$y^2\frac{dy}{dx} + \frac{1}{x}y^3 = \frac{1}{x}$

Let $w = y^3$

$\frac{dw}{dx} = 3y^2\frac{dy}{dx}$

$\frac{1}{3}\frac{dw}{dx} + \frac{1}{x}w = \frac{1}{x}$

$\frac{dw}{dx} + \frac{3}{x}w = \frac{3}{x}$ —(1)

$P(x) = \frac{3}{x}$

$I.F = e^{\int P(x)\,dx}$

$= e^{\int \frac{3}{x}dx}$

$= e^{3\ln x} = x^3$

Multiply (1) by x^3, we get

$x^3\frac{dw}{dx} + \frac{3}{x}x^3w = 3x^2$

$\frac{d}{dx}[x^3w] = 3x^2$

Integrating both sides

$x^3w = \int 3x^2\,dx$

$x^3w = x^3 + c$

$w = 1 + cx^{-3}$

$\boxed{y^3 = 1 + \frac{c}{x^3}}$

6. $(y\cos x + 2x)dx + (2y + \sin x)dy = 0$

$M = y\cos x + 2x$, $N = 2y + \sin x$

$\frac{\partial M}{\partial y} = \cos x$, $\frac{\partial N}{\partial x} = \cos x$

The solution is

$\int (y\cos x + 2x)dx$

$y = \text{const.}$

$+ \int 2y\,dy = c$

$y\sin x + x^2 + y^2 = c.$